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Development of a reduced relaxation function and comparison of stress relaxation for anatomically paired tendons †

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Abstract

A new "standard nonlinear solid" reduced relaxation function has been developed to describe the measured relaxation responses of tendons with 3% and 4% strain levels. This new reduced relaxation function has been used in modeling tendon responses with the quasi-linear viscoelastic law. Unlike the reduced relaxation functions employed in previous studies, the present function closely fits the measured relaxation responses for both the short term of the first few seconds and the long term of 22 hours. The relaxation responses of the anatomically paired tendons were found to be more alike than those from different sites.

Keywords: Anatomically paired; Strain level; Stress relaxation; Tendon; Viscoelastic law

1. Introduction

For many different biological tissues, a hereditary integral form of the stress-strain constitutive equation [1-4] has been used. This type of equation has been proposed by Fung [5,6] as the quasi-linear viscoelastic law (QVL). One of the important parts of this QVL is the reduced relaxation function. An exponential expression [2, 3] descriptive of a standard linear solid viscoelastic model and a logarithmic expression [1, 4] have been used for the reduced relaxation function thus far. However, these expressions have not described well the viscoelastic nature of biological tissues for both their short- and long-term relaxation responses. Also, the logarithmic expression predicts the negative (compressive) values of the relaxation function with a large time, which cannot be measured during relaxation tests.

The responses of the tendons are known to differ with the anatomical site [7-9]. In this study of tendon relaxation, the responses of anatomically paired tendons were examined to evaluate their similarity as a basis for evaluation differences due to other factors such as the strain level of the relaxation tests. The objectives of this work are twofold. The first is to compare the stress relaxation responses of the anatomically paired tendon specimens. The second is to develop a new reduced relaxation function to describe well the viscoelastic nature of the tendon.

2. Materials and methods

Anatomically paired tendon specimens were obtained from the hindlimbs of two canines which had been sacrificed in a veterinary surgery class. Within an hour post-mortem, the whole limbs were refrigerated at near freezing, then the tendons were dissected within a day. After dissection, each paired tendon specimen was wrapped in a Ringer's lactate-soaked paper towel and was sealed in a small plastic bag with the name of the anatomical location and the date of

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Table 1. Tendon specimen characteristics in relaxation test.

Strain Level	Test Number	Anatomical Site	Pair Status	Initial Length (mm)	Area (mm^2)	Initial Load (N)	Final Load (N)
3%	3-1	Extensor dig. long.	pair	34.26	0.85	12.1	2.1
	3-2	Extensor dig. long.		33.29	0.88	11.9	3.5
	3-3	Peroneus long.	pair	33.10	1.33	10.7	2.9
	3-4	Peroneus long.		32.45	2.22	7.7	1.8
	3-5	Extensor dig. long.	pair	32.04	2.06	10.3	3.4
	3-6	Extensor dig. long.		32.88	2.00	11.5	3.6
4%	4-1	Flexor dig. long.	pair	33.38	1.03	17.4	7.8
	4-2	Flexor dig. long.		33.43	1.21	23.7	10.5
	4-3	Extensor dig. long.	pair	35.03	1.63	22.6	4.1
	4-4	Extensor dig. long.		33.40	0.68	21.8	3.6
	4-5	Extensor dig. long.	pair	33.08	2.00	17.4	5.5
	4-6	Extensor dig. long.		32.75	1.33	12.8	3.0

※ dig. : digitorum long. : longus

dissection. Thereafter, groups of paired tendons from each canine were put into a larger plastic bag, then these were placed into a air-tight container and stored in a freezer at -70℃. This packing method prevented tendon dehydration and decay during storage. Each specimen was 45 mm or longer with a near constant cross-sectional area. Thick specimens with major or diameters larger than 3 mm were avoided because it was assumed that large cross-sections would not insure the uniform pressure between the interior and exterior fibers during gripping.

At the beginning of each test, a tendon specimen was soaked in Ringer's lactate solution for a minimum of 30 minutes during which there was complete thawing. One of the paired tendons was kept for the next test in a freezer. Tests were conducted with a servo-hydraulic Instron testing system. For gripping the tendon samples, flat-plate clamp type grips were employed with a waterproof 100 grit silicon carbide abrasive paper in the inner surface. Both sides of each end of all specimens were marked with a water-resistant pen, and these marks were placed inside the grips. These marks were observed and photographed with a WILD MPS 55 stereo-microscope and its camera during extensions. The marks were not detected by the microscope and shown by photographs, indicating that no detectable slippage occurred with this gripping method.

Grip motion was measured with an LVDT mounted in the hydraulic actuator of the Instron testing machine. The load was measured with a fully submersible Interface load cell (Model SSM-A5-1OO) which has a maximum of 100 lb load. The initial length of the specimen was measured between grips by a micrometer with an accuracy of 0.01 mm at a preload of 0.13 N. Relaxation tests were performed at strain levels of either 3% or 4% which were held constant during 22 hours. The specimens were initially stretched to these peak strains at a rate of 75 %/sec. The decrease in load for each test was checked at 21.5 hours after starting, and the test was finished at 0.5 hour later for a total test time of 22 hours. There was no decrease in load during the last half hour of any relaxation test. During testing, the specimen was immersed in Ringer's lactate solution bath at room temperature (22℃). Data were monitored and stored on a Nicolet digital oscilloscope (Model 201, Series 2090) and were transferred to a computer for analysis.

The cross-sectional areas of the samples were measured from histological cross-sections prepared with commonly used Paraffin embedding [10]. The slides with the cross-sections were placed in a photographic enlarger with a precision scale, and photographs were taken. From these photographs, the compact tendon cross-section was selected and measured using a Numonics digitizer which was accurate to within 0.01 mm^2. The strain level, test number, anatomical site, paired status, area, initial length, initial load, and final load for each specimen are listed in Table 1. Here, the initial load is the peak load when the specimen was stretched at the peak strain level, and the final load is the load when no more load relaxation occurred.

3. Results and discussion

3.1 Comparison of stress relaxation for the anatomically paired tendons

Table 2 shows the initial and final stresses for each specimen tested. Here, the initial stress is the peak stress when the specimen was stretched at the peak strain level, and the final stress is the stress when no more stress relaxation occurred.

Fig. 1(a) through 1(c) and Fig. 2(a) through 2(c) show the stress relaxation results of the paired tendon tests at 3% and 4% strain levels, respectively. The stress values have been normalized to a value of 1.0 for the initial stress value for comparison between the specimens and the interpretation as a reduced relaxation function. The stresses decreased quickly during the initial period of test time then continued to decrease at a slower rate. To show the significant stress relaxation in the presentation, the first initial 320 sec responses are shown in the above figures, since the stresses decreased very slowly after this time.

Table 2. Initial and final stress in the relaxation tests

Strain Level	Test Number	Initial Stress (MPa)	Final Stress (MPa)
3%	3-1	14.18	2.43
	3-2	13.57	4.02
	3-3	8.03	2.19
	3-4	3.48	0.82
	3-5	5.01	1.63
	3-6	5.77	1.79
4%	4-1	16.91	7.58
	4-2	19.57	8.69
	4-3	13.86	2.52
	4-4	31.99	5.30
	4-5	8.70	2.74
	4-6	9.63	2.26

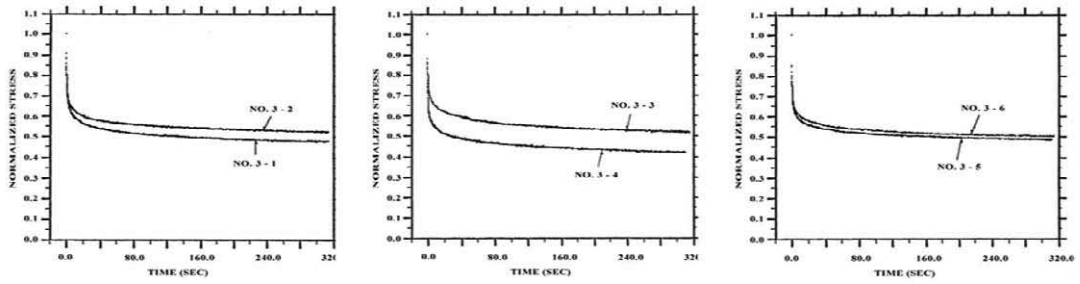
The tendon pairs 3-5 and 3-6 in Fig. 1(c) show similar responses. Almost the same stress relaxation responses are shown in Fig. 2(a) between the tendon pairs 4-1 and 4-2. However, the other tendon pairs as shown in Fig. 2(b) and 2(c) demonstrate different relaxation responses within a pair. Also, from Tables 1 and 2, specimens 3-1, 3-2, and 3-5, 3-6 are found to have similar areas and peak stresses. However, their stress relaxation responses are different as shown in Fig. 1(a) and Fig. 1(c). In the case of tests no. 4-1 and 4-2, the areas and peak stresses are different from one another, but the stress relaxation responses are almost the same.

3.2 Development of a reduced relaxation function

For the reduced relaxation function, an exponential expression [2, 3] for a standard linear solid viscoelastic model, $G(t) = a + be^{-ut}$, where a, b, and u are positive constants, and a logarithmic expression [1, 4], $G(t) = g - h\ell nt$, where g and h are positive constants, have been used thus far. However, the above expressions have not described well the viscoelastic nature of biological tissues. The logarithmic equation yields negative values with a large time and is not appropriate for modeling the long-term responses of soft tissues. The above exponential equation does not fit both short- and long-term responses.

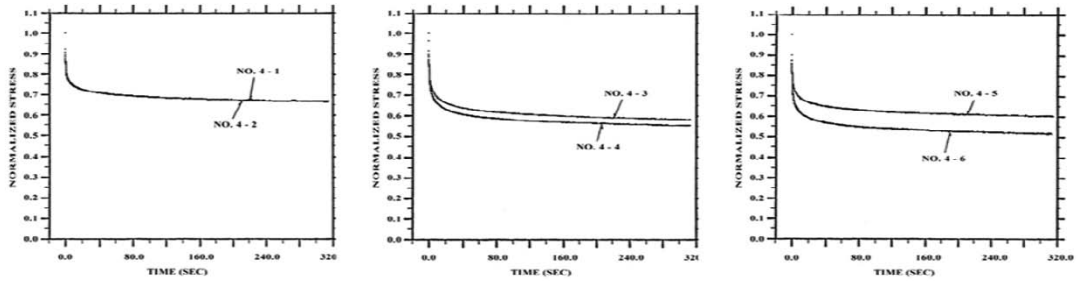
From the analysis of the relaxation test, a new reduced relaxation function for soft tissues is proposed in the following form:

$$G(t) = \alpha + \beta e^{-\mu t^{q}} \quad (1)$$



(a) (b) (c)

Fig. 1. Normalized stress relaxation of the paired tendons at 3% strain level tests; (a) test no. 3-1 and 3-2 (b) test no. 3-3 and 3-4, (c) test no. 3-5 and 3-6.



(a) (b) (c)

Fig. 2. Normalized stress relaxation of the paired tendons at 4% strain level tests; (a) test no. 4-1 and 4-2, (b) test no. 4-3 and 4-4, (c) test no. 4-5 and 4-6.

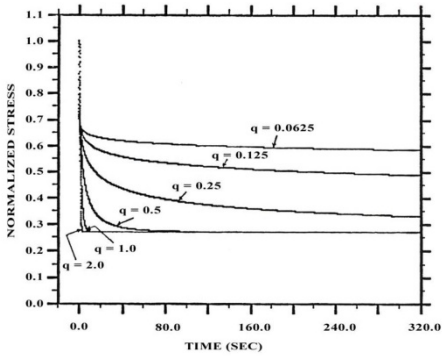


Fig. 3. The calculated data from the standard nonlinear solid reduced relaxation function with various q values.

It is called the standard nonlinear solid reduced relaxation function, where α, β, μ, and q are positive constant values. Fig. 3 shows the calculated data from Eq. (1) with parameters α=0.27, β=0.13, μ=0.59, and various q values. The effect of the value of q is shown in this figure.

Since G(t) for t = 0 is defined to have a value of 1.0, we obtain the following:

$$G(t) = \alpha + \beta = 1.0 \tag{2}$$

As t $\rightarrow \infty$, Eq. (1) tends to be α (positive value). Since the soft connective tissues do not behave like fluid but instead act as solid viscoelastic materials, then

$$\lim_{t\to\infty} G(t) \to \alpha > 0 \tag{3}$$

This is shown for the tendon above in this section by no further relaxation after 21.5 hours.

The differentiation of Eq. (1) yields the following:

Table 3. Summary of the coefficients in the reduced relaxation function for the paired tendons.

$$G(t) = \alpha + \beta e^{-\mu t^q}$$

Strain Level	Test Number	α	β	μ	q	Residual Sum
3%	3-1	0.17	0.83	0.49	0.130	0.99
	3-2	0.30	0.70	0.56	0.126	0.98
	3-3	0.27	0.73	0.48	0.144	0.34
	3-4	0.24	0.76	0.68	0.129	0.40
	3-5	0.33	0.67	0.71	0.124	0.94
	3-6	0.31	0.69	0.64	0.122	0.61
	Ave.	0.27	0.73	0.59	0.129	0.71
	S.D.	0.06	0.06	0.09	0.008	0.30
	95%CI	0.06	0.06	0.09	0.008	0.32
4%	4-1	0.45	0.55	0.46	0.128	0.59
	4-2	0.45	0.55	0.47	0.121	0.27
	4-3	0.18	0.82	0.35	0.131	0.57
	4-4	0.17	0.83	0.38	0.127	0.86
	4-5	0.31	0.69	0.46	0.116	0.42
	4-6	0.23	0.77	0.50	0.123	1.07
	Ave.	0.30	0.70	0.44	0.124	0.63
	S.D.	0.13	0.13	0.06	0.005	0.29
	95%CI	0.14	0.14	0.06	0.005	0.30

$$\frac{dG(t)}{dt} = \beta e^{-\mu t^q} \frac{d(-\mu t^q)}{dt}$$

$$= -\beta\mu q t^{q-1} e^{-\mu t^q} < 0 \tag{4}$$

where $\frac{dG(t)}{dt}$ is the slope of the reduced relaxation function.

Since α, β, and q are always positive, the slope of G(t) is negative. Thus, this reduced relaxation function is a continuously decreasing function of time. The tissue behavior satisfies the fading memory principle [11] with increasing time. This fading memory principle is reasonable because the tissue behavior would be physically unrealistic with a growing mem-

ory for the more distant action. In fact, all relaxation tests for the tendons in this study have satisfied this principle.

The coefficients of the reduced relaxation function were determined by a least square error method [12] and are listed in Table 3. The residual for a data point is the difference between the actual data value and its estimated value from an equation. The sum of squares of the residuals, the residual sum, estimates the quality of fit of the data to the linear regression equation. When the data are exactly linear, the residual sum is zero. The small individual value of the residual sum with the least-square technique shows a high quality of fit with about 2,000 data points for each linear regression equation. Also, this table presents the averages, standard deviations, and 95% confidence intervals from the t - test table [12].

For the 3% strain level, the value of α is statistically lower, and the values of β, μ, and q are statistically higher than those for the 4% strain level, and all values are positive and less than 1. Also, this table shows that the coefficients of the anatomically paired tendons are similar to one another with a few exceptions.

In Table 4, the significance of the statistical data is shown by comparing two (3% and 4%) population means at a 95% confidence interval with a t - test table [12]. Here, the critical t - value is ±2.228. This comparison shows that a significant difference occurs only for μ, and the other coefficients of the reduced relaxation function are not statistically different.

Figs. 4 and 5 show the normalized stress relaxations as measured and calculated from Eq. (1) in typical 3% (test no. 3-3) and 4% (test no. 4-5) strain level tests, respectively. The agreement between the measured data and the calculated data from the standard nonlinear solid reduced relaxation function is excellent and becomes better with a time up to 22 hours. However, in the presentation, these figures show the initial 320 sec data in presenting the significant stress relaxation which occurs during the initial test time.

Table 4. Comparison of two (3% and 4%) population means for the values of the coefficients at the 95% confidence interval with a t-test table.

α	β	μ	q
-0.513	0.513	3.397 *	1.298

Critical value : ±2.228
* : the value which is over the critical value

In summary, the anatomically paired tendons have similar stress relaxation responses. However, these similarities are not always consistent. A new reduced relaxation function called the standard nonlinear solid reduced relaxation function satisfies the fading memory principle and has an excellent agreement, which the previous models [1-4] could not achieve in the short- and long-term responses. This new function can be applied to the development of QVL and gives the QVL an excellent prediction for short- and long-term multiple cyclic tests [13].

The results of this study offer new information on the mechanical nature of collagenous tissues. We have learned more about their responses to long-term relaxation tests. The relaxation or reduction of resistance to extension is sought to increase musculoskeletal mobility in athletic training, manual medicine, or

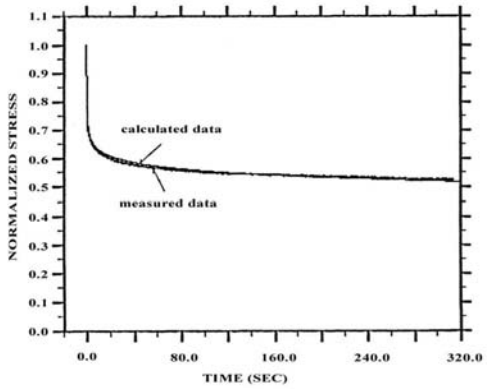


Fig. 4. Typical normalized stress relaxation at the 3% (test no. 3-3) strain level as measured and calculated with the standard nonlinear solid reduced relaxation function.

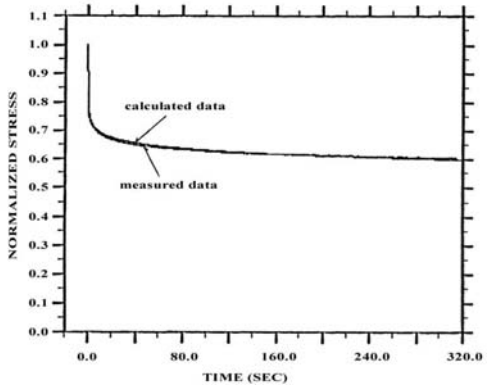


Fig. 5. Typical normalized stress relaxation at a 4% (test no. 4-5) strain level as measured and calculated with a standard nonlinear solid reduced relaxation function.

physical therapy. However, the nature of these relaxation phenomena in tissues needs to be explored further in future work so that we can better understand tissue responses and musculoskeletal function.

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